Economic and Reliability Analysis of a Centrifuge System with Rest period, Neglected Faults and Stoppage on Minor Faults

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ABSTRACT—The paper deals with a model developed for a single centrifuge system working in Thermal Power Plant, Panipat (Haryana) India, which has alternate periods of operation and rest. The system may have minor, neglected and major faults. It is assumed that the occurrence of a minor fault leads to degradation of the system whereas occurrence of a major fault leads to failure of the system. The neglected faults that are in the system are generally neglected for repair during operation of the system until the system goes to rest or complete failure and the system has to be stopped on occurrence of minor fault for repair. Various measures of system effectiveness are obtained regarding the reliability and cost analysis of the system is carried out and the conclusions on the basis of the graphical studies are given.

KEY WORDS—centrifuge system, mean time to system failure, neglected faults, Markov process, profit, regenerative point technique.

1 INTRODUCTION

IN the field of reliability modeling, several different types of systems considering various aspects such as types of failure (faults) repairs, inspection polices, modes of operations, switching etc. have been analyzed by several researchers including [1],[2],[3],[4],[5],[6], [7].

In many practical situations, for instance in thermal power plant for oil purification, milk plants for making butter, laboratories, blood fractionation, wine clarification, etc. centrifuge systems are used and act as the main systems or sub-systems. In these situations the reliability and cost of centrifuge systems play a very important and crucial role.

It was observed, while collecting real data on faults/ failures and repairs on a centrifuge system working in Thermal Power plant, Panipat (Haryana) which undergoes periodic rest (normally after eight hours), that a minor fault leads to degradation of the system whereas a major fault leads to complete failure of the system. Some faults such as vibration, abnormal sound, etc are generally neglected for repair during the operation of the system until system goes to rest or to complete failure. Sometimes these neglected faults also lead to complete failure of the system. Further the system has to be stopped on occurrence of minor fault. The cost and maintenance analysis of centrifuge system considering the aspects of periodic rest period, neglected faults and stoppage on occurrence of minor faults has not been reported in the literature of reliability so far. However, the reliability and availability analyses of a centrifuge system considering minor, ignored and major faults has been carried out by [8], [9].

Keeping above in view, the present paper deals with a single unit centrifuge system considering major, minor and neglected faults wherein a minor fault degrades the system whereas a major fault leads to complete failure of the system. The neglected fault is taken as the fault that may be neglected for repair during the operation of the system until system goes to rest or to complete failure. The system undergoes periodic rest. During the rest period or complete failure, the repairman first inspect whether the fault is repairable or non repairable and accordingly carry out repair or replacement of the faulty components. Sometimes when minor fault occurs, the system has to be stopped. Various measures of system effectiveness, such as mean sojourn times, mean time to system failure, expected uptime, busy period of repairman and expected profit are obtained using Markov processes and regenerative point technique. Various conclusions regarding the reliability and cost of the system on the basis of graphical analyses are drawn.

2 OTHER ASSUMPTIONS

1. Faults are self- announcing.
2. There is single repairman that reaches the system in negligible time, whenever called for repair.
3. The system is as good as new after each repair / replacement.
4. Switching is perfect and instantaneous.
5. The time distributions of various faults, rest, stoppage and restart of the system are exponential whereas other time distributions are general.

3 NOTATIONS

$\lambda_1/\lambda_2/\lambda_3$	Rate of occurrence of major/minor/ neglected faults
a/b	Probability that the fault is non repairable / repairable, b = 1- a

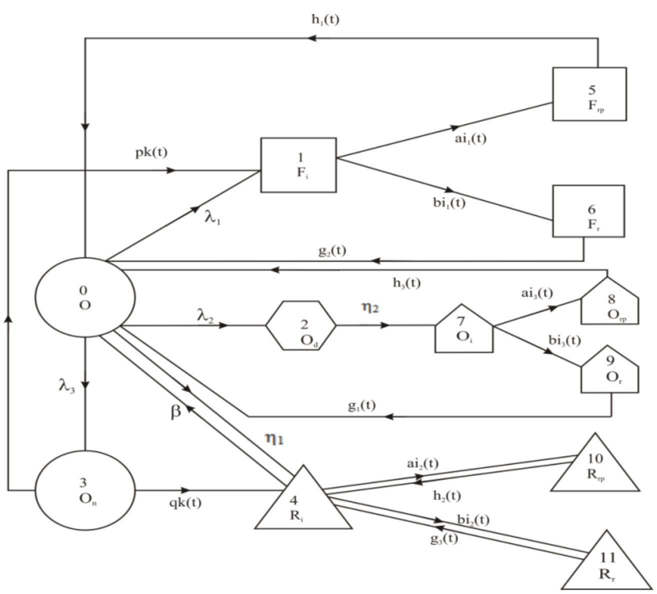
p/q	Probability that the neglected fault lead to/ don't lead to complete failure, q=1-p
η_1	Rate at which the unit goes to rest
η_2	Rate at which the unit is stopped
β	Rate at which the unit restarts after rest
$i_1(t)/i_2(t)/i_3(t)$	p.d.f. of time to inspection of the unit at failed/stopped/rest state
$I_1(t)/I_2(t)/I_3(t)$	c.d.f. of time to inspection of the unit at failed/stopped/rest state
$g_1(t)/g_2(t)/g_3(t)$	p.d.f. of time to repair the unit at failed / stopped/rest state
$G_1(t)/G_2(t)/G_3(t)$	c.d.f. of time to repair the unit at failed / stopped/rest state
$h_1(t)/h_2(t)/h_3(t)$	p.d.f. of time to replacement of the unit at failed/stopped/rest state
$H_1(t)/H_2(t)/H_3(t)$	c.d.f. of time to replacement of the unit at failed/stopped/rest state
$k(t)/K(t)$	p.d.f./c.d.f. of time to delay in repair of the neglected fault

3.1 STATES OF THE SYSTEM

O/R	Operative / Rest state
$O_r/O_n/O_d$	Operative state under inspection/ neglected fault/degradation
$R_i/R_r/R_{rp}$	Rest state under inspection/repair/ replacement
$F_i/F_r/F_{rp}$	Failed state under inspection/ repair/ replacement

4 THE MODEL

A diagram showing the various states of transition of the system is shown in Fig. 1. The epochs of entry in to state 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 are regenerative point and thus all the states are regenerative states.



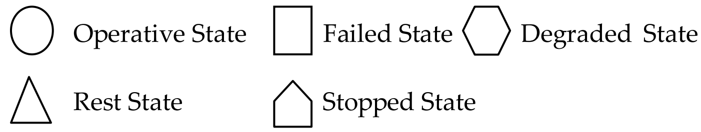


Fig.1 State Transition Diagram

5 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The transition probabilities are

$$dQ_{01}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2+\lambda_3+\eta_1)t}dt \qquad dQ_{02}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2+\lambda_3+\eta_1)t}dt$$
$$dQ_{03}(t) = \lambda_3 e^{-(\lambda_1+\lambda_2+\lambda_3+\eta_1)t}dt \qquad dQ_{04}(t) = \alpha e^{-(\lambda_1+\lambda_2+\lambda_3+\eta_1)t}dt$$
$$dQ_{15}(t) = ai_1(t)dt \qquad dQ_{16}(t) = bi_1(t)dt$$
$$dQ_{27}(t) = \eta_2 e^{-\eta_2(t)}dt \qquad dQ_{31}(t) = pk(t)dt$$
$$dQ_{34}(t) = qk(t)dt \qquad dQ_{40}(t) = \beta e^{-\beta(t)}\overline{I_2}(t)dt$$
$$dQ_{4,10}(t) = ai_2(t)e^{-\beta(t)}dt \qquad dQ_{4,11}(t) = bi_2(t)e^{-\beta(t)}dt$$
$$dQ_{50}(t) = h_1(t)dt \qquad dQ_{60}(t) = g_2(t)dt$$
$$dQ_{78}(t) = ai_3(t)dt \qquad dQ_{79}(t) = bi_3(t)dt$$
$$dQ_{80}(t) = h_3(t)dt \qquad dQ_{90}(t) = g_1(t)dt$$
$$dQ_{10,4}(t) = h_2(t)dt \qquad dQ_{11,4}(t) = g_3(t)dt$$

The non-zero elements p_{ij} are $p_{ij} = \lim_{s\to 0} Q_{ij}^{**}(s)$

$$p_{01} = \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3+\eta_1} \qquad p_{02} = \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3+\eta_1}$$
$$p_{03} = \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3+\eta_1} \qquad p_{04} = \frac{\eta_1}{\lambda_1+\lambda_2+\lambda_3+\eta_1}$$
$$p_{15} = ai_1^*(0) \qquad p_{16} = bi_1^*(0)$$
$$p_{31} = pk^*(0) \qquad p_{34} = qk^*(0)$$
$$p_{40} = 1 - i_2^*(\beta) \qquad p_{4,10} = bi_2^*(\beta)$$
$$p_{4,11} = ai_2^*(\beta) \qquad p_{50} = h_1^*(0)$$
$$p_{60} = g_2^*(0) \qquad p_{78} = ai_3^*(0)$$
$$p_{79} = bi_3^*(0) \qquad p_{80} = h_3^*(0)$$
$$p_{90} = g_1^*(0) \qquad p_{10,4} = h_2^*(0)$$
$$p_{11,4} = g_3^*(0)$$

By these transition probabilities, it can be verified that $p_{01} + p_{02} + p_{03} + p_{04} = p_{15} + p_{16} = p_{34} + p_{31} = p_{4,10} + p_{4,11} + p_{40} = p_{78} + p_{79} = 1$, $p_{27} = p_{50} = p_{60} = p_{80} = p_{90} = p_{10,4} = p_{11,4} = 1$

The mean sojourn time in the regenerative state $i(\mu_i)$ is defined as the time of stay in that state before transition to any other state then we have

$$\mu_0 = \frac{1}{\lambda_1+\lambda_2+\lambda_3+\eta_1} \qquad \mu_1 = -i_1^{*\prime}(0) \qquad \mu_2 = \frac{1}{\eta_2}$$

$\mu_3 = -k^{*\prime}(0)$ $\quad \mu_4 = \frac{1-i_2^*(\beta)}{\beta}$ $\quad \mu_5 = -h_1^{*\prime}(0)$

$\mu_6 = -g_2^{*\prime}(0)$ $\quad \mu_7 = -i_3^{*\prime}(0)$ $\quad \mu_8 = -h_3^{*\prime}(0)$

$\mu_9 = -g_1^{*\prime}(0)$ $\quad \mu_{10} = -h_2^{*\prime}(0)$ $\quad \mu_{11} = -g_3^{*\prime}(0)$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as-

$$m_{ij} = \int t\, dQ_{ij}(t)$$

Thus,

$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$ $\quad m_{15} + m_{16} = \mu_1$

$m_{27} = \mu_2$ $\quad m_{34} + m_{31} = \mu_3$

$m_{40} + m_{4,10} + m_{4,11} = \mu_4$ $\quad m_{50} = \mu_5$

$m_{60} = \mu_6$ $\quad m_{78} + m_{79} = \mu_7$

$m_{80} = \mu_8$ $\quad m_{90} = \mu_9$

$m_{10,4} = \mu_{10}$ $\quad m_{11,4} = \mu_{11}$

6 Other Measures of System Effectiveness

Using probabilistic arguments for regenerative processes, various recursive relations are obtained and are solved to derive important measures of the system effectiveness that are as given below:

Mean time to system failure

$(T_O) = \mu_0 + \mu_2 p_{02}$

Expected up time of the system

$(A_O) = N_1/ D_1$

Busy period of repair man (Inspection time only)

$(B_i) = N_2/ D_1$

Busy period of repair man (Repair time only)

$(B_r) = N_3/ D_1$

Busy period of repair man (Replacement time only)

$(B_{rp}) = N_4/ D_1$

where

$N_1 = \mu_0 + \mu_2 p_{02} + \mu_3 p_{03}$

$N_2 = p_{40}\,[p_{02}p_{27}\,\mu_7 + (p_{01}+p_{03}p_{31})\,\mu_1 + (p_{03}p_{34}+p_{04})\,\mu_4]$

$N_3 = p_{40}[p_{02}\mu_2 + p_{16}\,\mu_6(p_{01}+p_{03}p_{31}) + p_{02}p_{27}p_{79}\,\mu_9]$
$\quad + p_{4,11}\mu_{11}(p_{03}p_{34}+p_{04})$

$N_4 = p_{40}[p_{02}p_{27}p_{78}\,\mu_8 + p_{15}\mu_5(p_{01}+p_{03}p_{31})] + \mu_{10}p_{4,10}(p_{03}p_{34}+p_{04})$

$D_1 = p_{40}\,[\mu_0 + p_{03}\,\mu_3 + p_{02}\,(\mu_2 + p_{79}\mu_9 + p_{78}\mu_8 + \mu_7) +$
$(p_{01}+p_{03}p_{31})\,(\mu_1 + p_{15}\,\mu_5 + p_{16}\mu_6)] +$
$(\mu_{11}p_{4,11} + \mu_{10}p_{4,10} + \mu_4)\,(1-p_{01}-p_{02}-p_{03}p_{31})$

7 PROFIT ANALYSIS

The expected profit incurred of the system is

$P = C_0\, A_0 - C_1\, B_i - C_2\, B_r - C_3 B_{rp} - C$

where

C_0 = revenue per unit uptime of the system

C_1 = cost per unit time of inspection

C_2 = cost per unit time of repair

C_3 = cost per unit time of replacement

C = cost of installation of the unit

8 GRAPHICAL ANALYSES

For graphical analysis the following particular cases are considered:

$g_1(t) = \beta_1 e^{-\beta_1(t)}$ $\quad g_2(t) = \beta_2 e^{-\beta_2(t)}$ $\quad g_3(t) = \beta_3 e^{-\beta_3(t)}$

$k(t) = \delta e^{-\delta(t)}$ $\quad h_1(t) = \gamma_1 e^{-\gamma_1(t)}$ $\quad h_2(t) = \gamma_2 e^{-\gamma_2(t)}$

$h_3(t) = \gamma_3 e^{-\gamma_3(t)}$ $\quad i_1(t) = \alpha_1 e^{-\alpha_1(t)}$ $\quad i_2(t) = \alpha_2 e^{-\alpha_2(t)}$

$i_3(t) = \alpha_3 e^{-\alpha_3(t)}$

Various graphs are drawn for the MTSF and the profit (P) for the different values of the rates of occurrence of major, minor and neglected faults (λ_1, λ_2, λ_3), repair rates (β_1, β_2, β_3), replacement rates (γ_1, γ_2, γ_3), inspection rates (α_1, α_2, α_3) and rates of rest, stoppage and delay for repair (η_1,η_2,δ) of the unit.

Fig. 2 and Fig.3 give the graphs between MTSF (To) and the rate of occurrence of major and neglected faults (λ_1 & λ_3), respectively for different values of rate of occurrence of minor faults (λ_2) and rate at which the system has to be stopped (η_2). The graph reveals that the MTSF deceases with increase in the values of rates of occurrence of major, minor and neglected faults, respectively. Further it may also be concluded from the Fig.3 that the delay in the repair of neglected faults result into decrease in the values of MTSF.

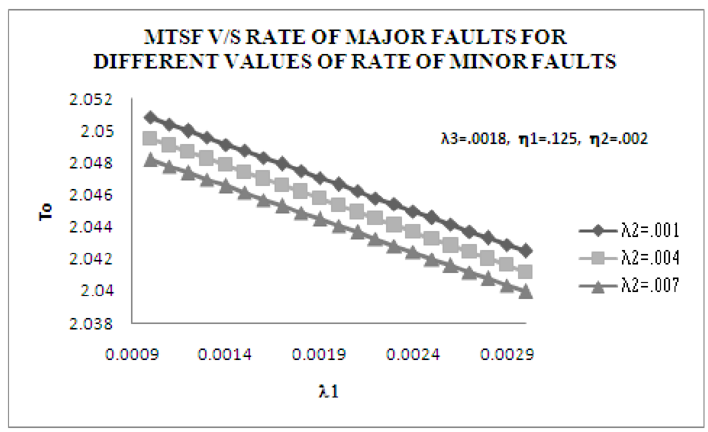


Fig. 2

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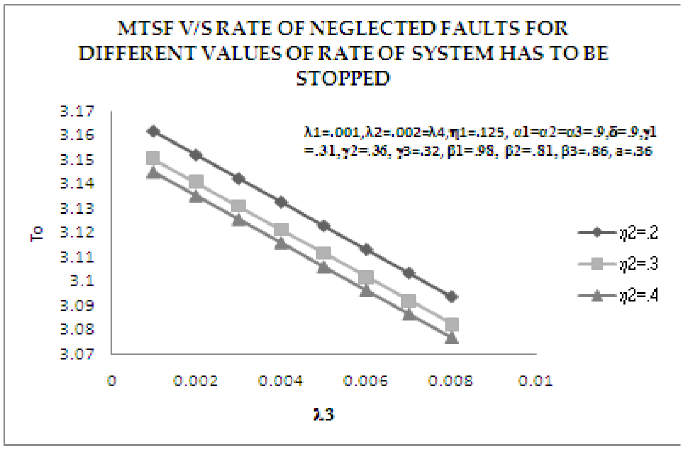

Fig.3

The graph in Fig. 4 shows the pattern of profit with respect to rates of occurrence of minor faults for different values of major faults (λ_2 & λ_1). The curves in the graph indicate that the profit of the system decreases with the increase in the values of the rates of occurrence of minor faults for different values of major faults.

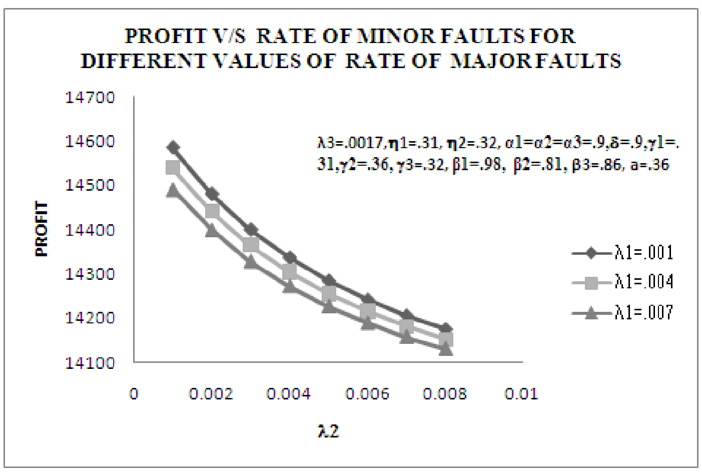

Fig.4

The curves in the Fig.5 show the behavior of the profit with respect to rate of occurrence of neglected faults (λ_3) of the system for the different values of rate of delay in repair of neglected faults (δ). It is evident from the graph that profit decreases with the increase in the rate due to occurrence of neglected faults. From the Fig. 5 it may also be observed that for δ =2, the profit is > or = or < 0 according as λ_3 is < or = or >.879. Hence the system is profitable to the company whenever $\lambda_3 \leq .879$. Similarly, for δ =3 and δ =4 respectively the profit is > or = or < 0 according as λ_3 is < or = or > .825 and .8 respectively. Thus, in these cases, the system is profitable to the company whenever $\lambda_3 \leq .825$ and .8 respectively.

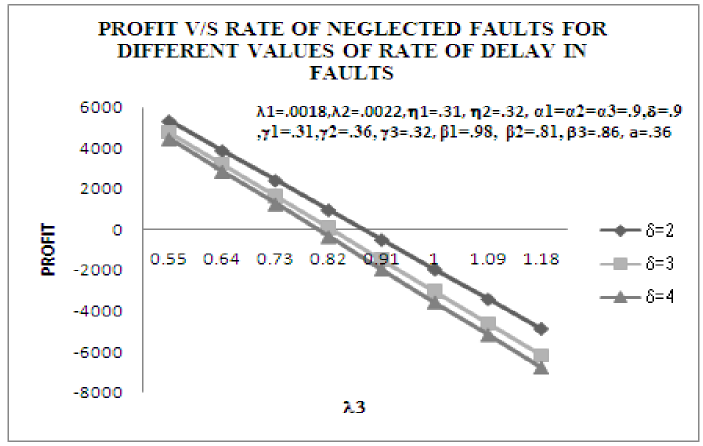

Fig. 5

The graph in Fig. 6 shows the pattern of profit with respect to the rates of occurrence of neglected faults for different values of rate of system has to be stopped (λ_3,η_2). The curves in the graph indicate that the profit of the system decreases with the increase in the values of the rate of occurrence of neglected faults as well as rate with which system is stopped.

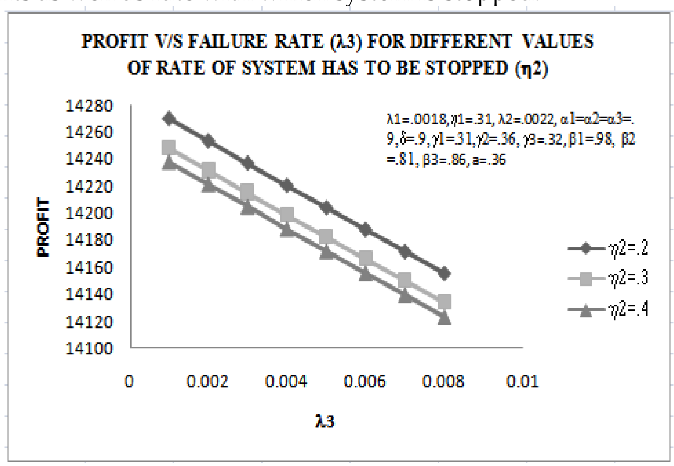

Fig. 6

The curves in the Fig.7 show the behavior of the profit with respect to the revenue per unit up time (C_0) of the system for the different values of rate of occurrence of neglected faults (λ_3) due to neglected faults. It is evident from the graph that profit increases with the increase in revenue up time of the system for fixed value of the rate of occurrence of neglected faults. From the Fig.7 it may also be observed that for λ_3 = 0.001, the profit is > or = or < 0 according as C_0 is > or = or < Rs.20664.21. Hence the system is profitable to the company whenever $C_0 \geq$ Rs. 20664.21. Similarly, for λ_3 = 0.201 and λ_3 = 0.401 respectively the profit is > or = or < 0 according as C_0 is > or = or < Rs.24093.56 and Rs.28257.06 respectively. Thus, in these cases, the system is profitable to the company whenever $C_0 \geq$ Rs. 24093.56 and Rs. 28257.06 respectively.

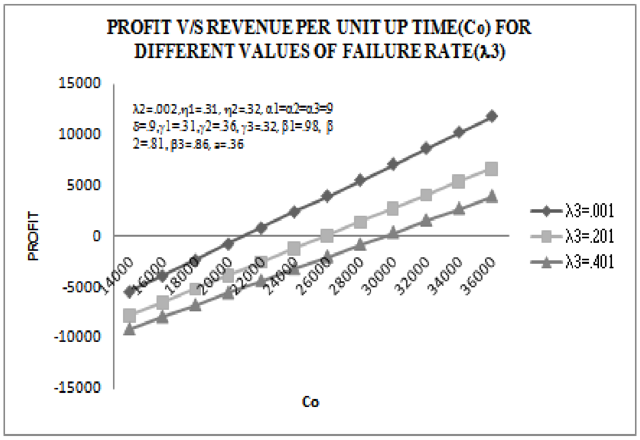

Fig. 7

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